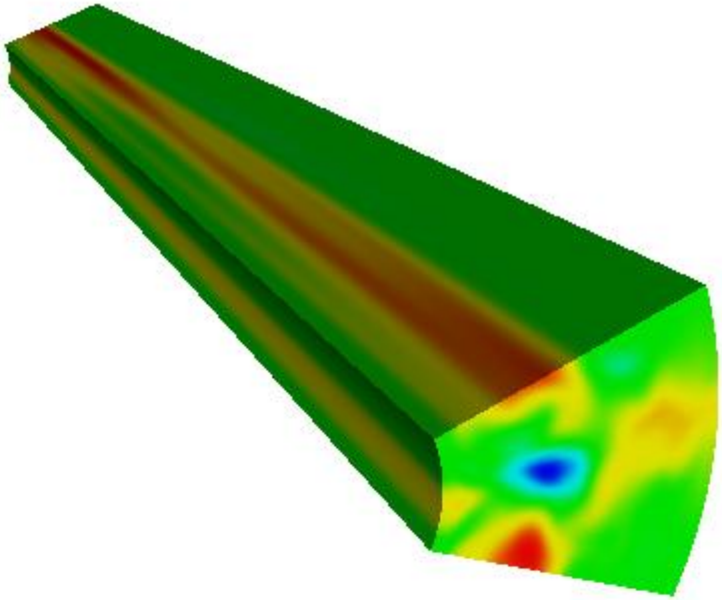
BOUT++ Simulation of LAPD Turbulence

BOUT++ Workshop 2011



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[1]LLNL

Outline

I. The LAPD fluid model and LAPD geometry

II. Linear BOUT++ instability analysis and verification against eigenvalue solver

III. Nonlinear simulations and comparison to experiment

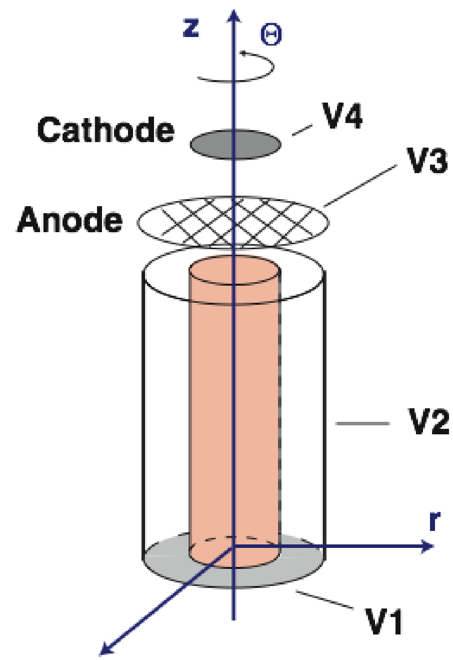
IV. Grid resolution study

V. Initial work on axial sheath boundary conditions

VI. Conclusions

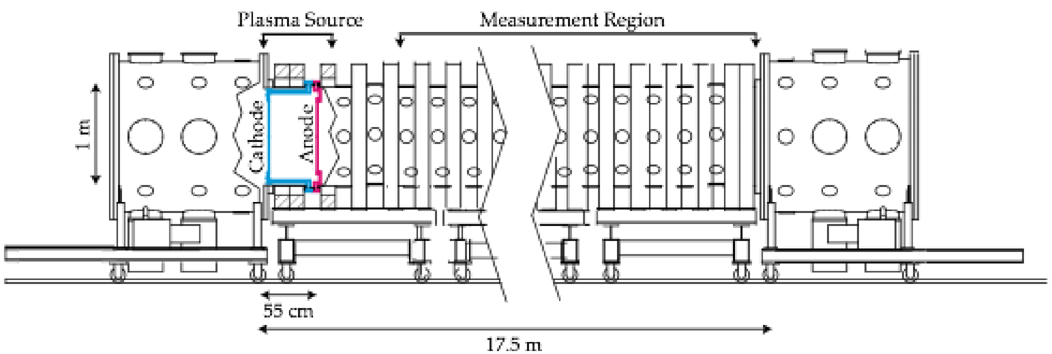
LAPD is Ideal for Collisional Plasma Fluid Model

Machine and plasma size:
Plasma column length $\sim 18\ m$
Plasma radius $\sim 30\ cm$



Typical LAPD operational parameters:
$0.4 < B_0 < 2\ kG$
$10^{11} < n_e < 4 \times 10^{12}\ cm^{-3}$
$0.5 < T_e < 8\ eV$
$0.5 < T_i < 1.5\ eV$
$f_{ci} \sim 400\ KHz$
$\nu_{in} \sim 2\ KHz$
$\nu_{ei} \sim 5\ MHz$
$\frac{\omega}{k_\parallel} \leq v_{the}$

$\lambda_{ei}/L_\parallel \sim 0.01$
$\nu_i/\omega_{ci} \sim 1$
$k_\perp \rho_i \sim 0.1$



LAPD Model: Three Field Drift Wave Model (lapd-drift)

Three-field electrostatic model implemented

$$\frac{\partial N_i}{\partial t} + \nabla \cdot (N_i \mathbf{v}) = S_p$$

$$N_i m \frac{d\mathbf{v}_e}{dt} = -\nabla p - N_i e \left(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}\right) - N_i m_e \nu_{ei} (\mathbf{v}_e - \mathbf{v}_i) - N_i m_e \nu_{en} \mathbf{v}_e$$

$$\nabla \cdot \mathbf{J} = 0 \quad \text{where} \quad \nabla \cdot \mathbf{J}_\perp = \nabla \cdot e N_i (\mathbf{v}_{i\perp} - \mathbf{v}_{e\perp}) = \nabla \cdot \frac{c^2 m_i N_i}{B^2} \left(\frac{d\mathbf{E}_\perp}{dt} + \nu_{in} \mathbf{E}_\perp \right)$$

BOUT++ Equations Solved

$$\frac{\partial N}{\partial t} = -\mathbf{v}_E \cdot \nabla_\perp N - \nabla_\parallel (N v_{\parallel e}) + D \nabla_\perp^2 N + S_p$$

$$\frac{\partial v_{\parallel e}}{\partial t} = -v_{\parallel e} \nabla_\parallel v_{\parallel e} - \frac{T_e}{N} \nabla_\parallel N + N \nabla_\parallel \phi - 0.51 \nu_e N v_{\parallel e}$$

$$\partial_t \varpi = -\mathbf{v}_E \cdot \nabla \varpi + \nabla_\parallel (N v_\parallel) + \frac{1}{2} (\mathbf{b} \times \nabla N) \cdot \nabla_\perp \mathbf{v}_E^2 - \nu_{in} \varpi + \mu \nabla_\perp^2 \varpi$$

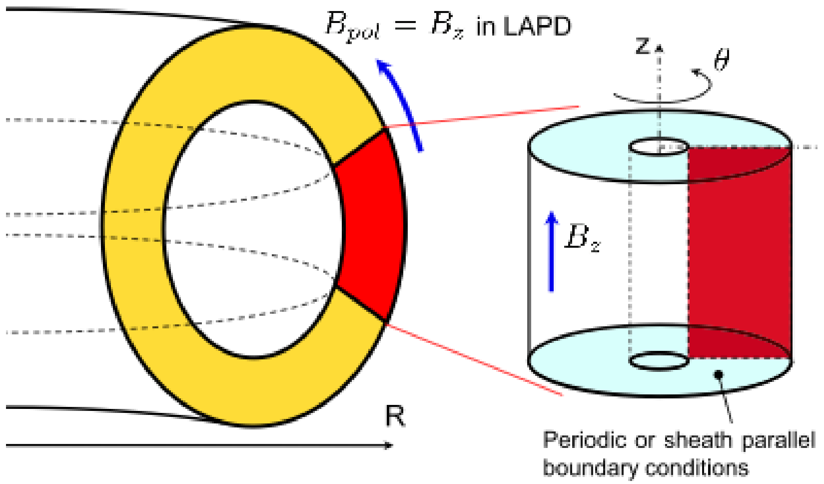
$$\varpi \equiv \nabla_\perp \cdot (N \nabla_\perp \phi)$$

Instabilities Supported with these Equations

- Resistive drift waves
- Kelvin-Helmholtz
- Rotational Interchange

Geometry and Boundary Conditions Used in Standard LAPD Simulation

Geometry in Simulation is Cylindrical Annulus



$$x \rightarrow r$$
$$y \rightarrow z \text{ (poloidal)}$$
$$z \rightarrow \theta \text{ (toroidal, periodic)}$$

Grid file created using IDL program written by M.V. Umansky and P. Popovich input into modified UEDGE grid generator

- Periodic axial boundaries
- Zero-derivative (Neumann) radial boundaries
- Radial potential b.c. used in vorticity inversion:

Inner radial boundary: $$\frac{\partial \phi_{DC}}{\partial r} = \phi_{AC} = 0$$

Outer radial boundary: $$\phi_{DC} = \frac{\partial \phi_{AC}}{\partial r} = 0$$

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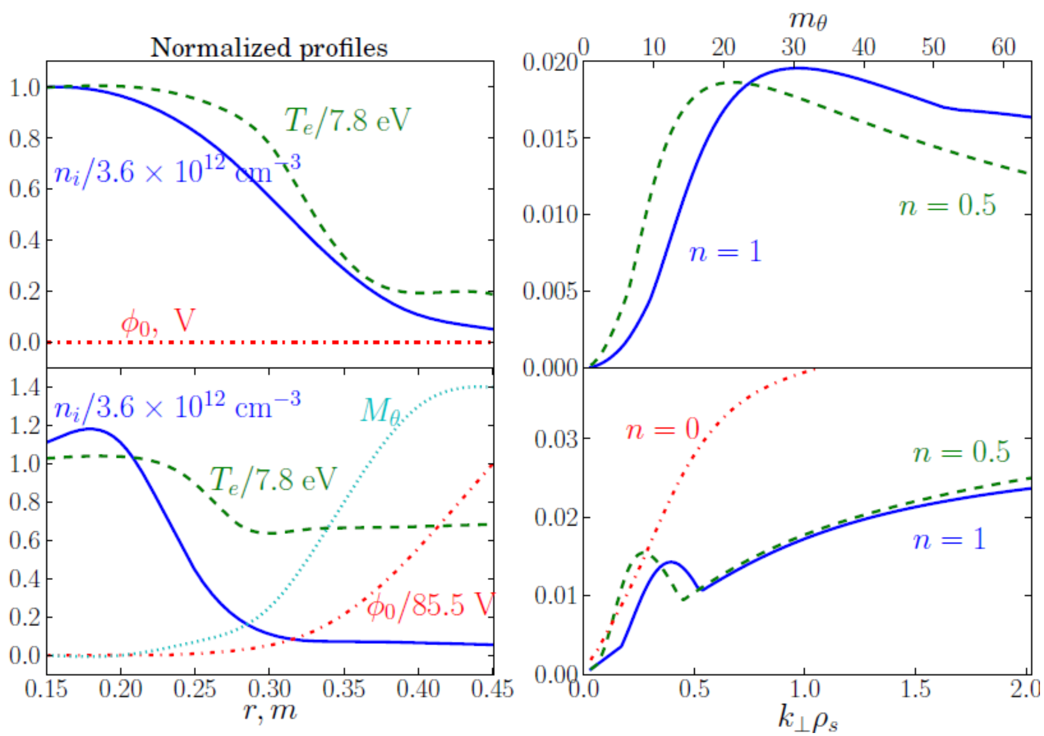
Linear Instability Analysis Done with Eigenvalue Solver*

Fields $N, v_{\|e}, \phi$ Fourier decomposed: $F(r,\theta,z,t) = f(r)exp(im\theta + ik_z z - i\omega t)$

Generalized eigenvalue problem: $-i\omega \mathbf{A}\mathbf{v} = \mathbf{B}\mathbf{v}, \quad \mathbf{v} = \begin{pmatrix} n(r) \\ v_{\|e}(r) \\ \phi(r) \end{pmatrix}$

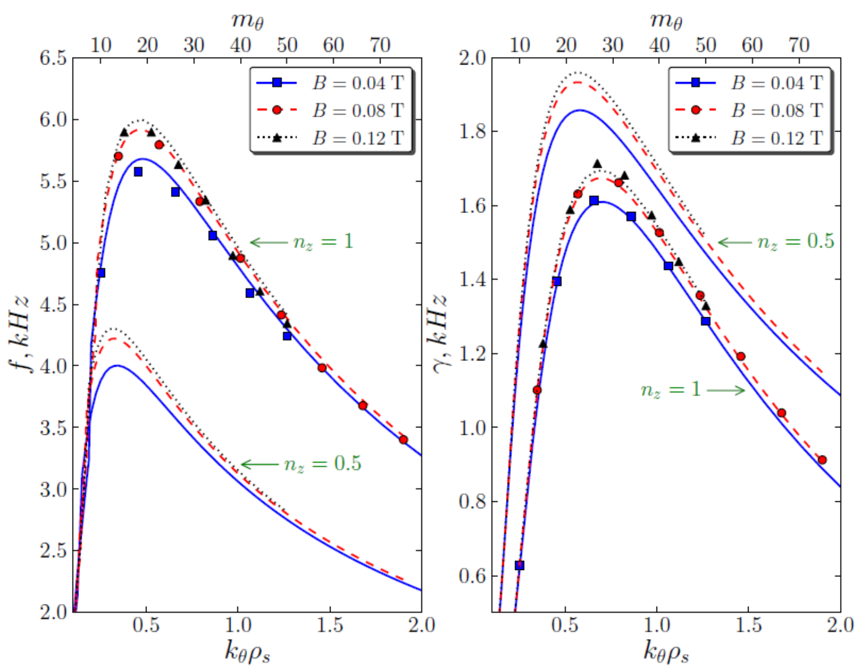
- Drift waves, KH, Interchange instabilities explored with different equilibrium profiles and parallel wave numbers.
- Linear growth rates for most unstable eigenvalue shown.



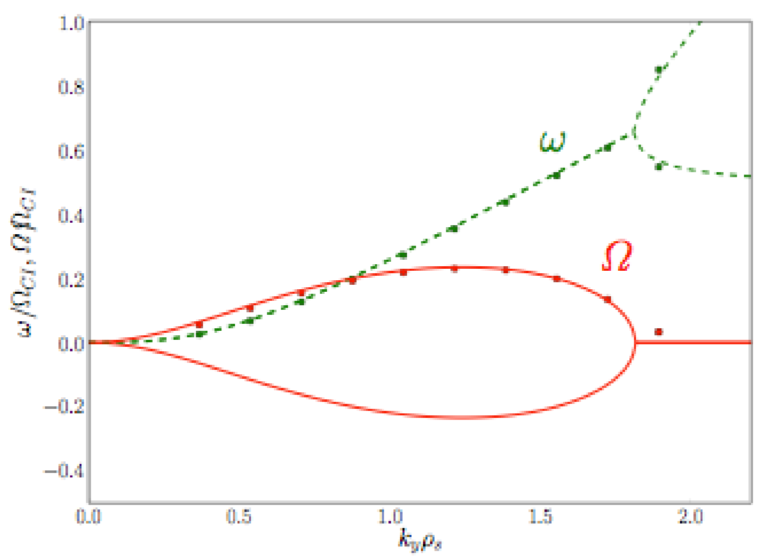
* P Popovich, B Friedman. Open source python code on google codes. Called Braginskii eigsolver

BOUT++ First Verified Against Eigenvalue Solver

Drift wave dispersion relation and growth rates for fastest growing radial eigenmode (curves) vs BOUT++ (dots) result. LAPD experimental density profile.



KH test case potential profile with both branches of analytic solution (curves) vs BOUT++ result.



*Linear analysis and BOUT verification:
P. Popovich, M.V. Umansky, T.A. Carter, B. Friedman (2010)

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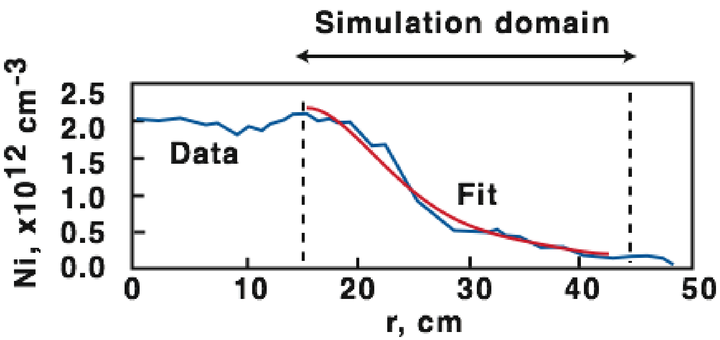
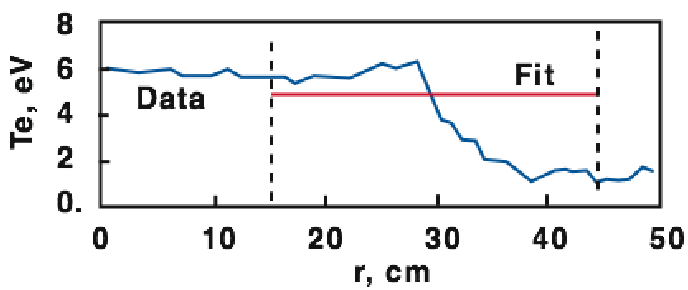
Profiles Used in Simulation Compared to Experiment

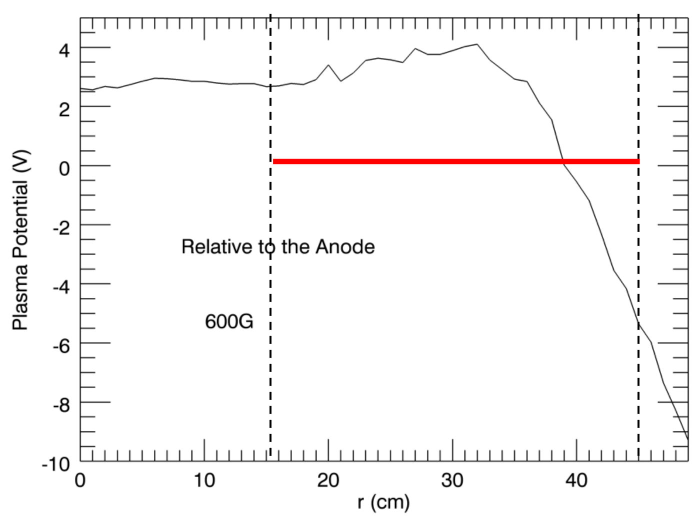


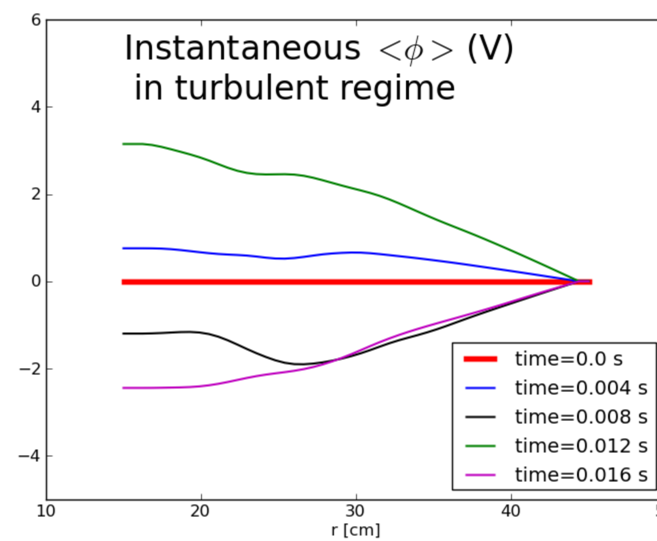


- Density equilibrium profile fit to experiment.
- Density source used to keep average density equal to equilibrium – subtracts m=0 density fluctuation component.

- T_e = 5eV   constant profile
- T_i = 0eV
- No temperature fluctuations.

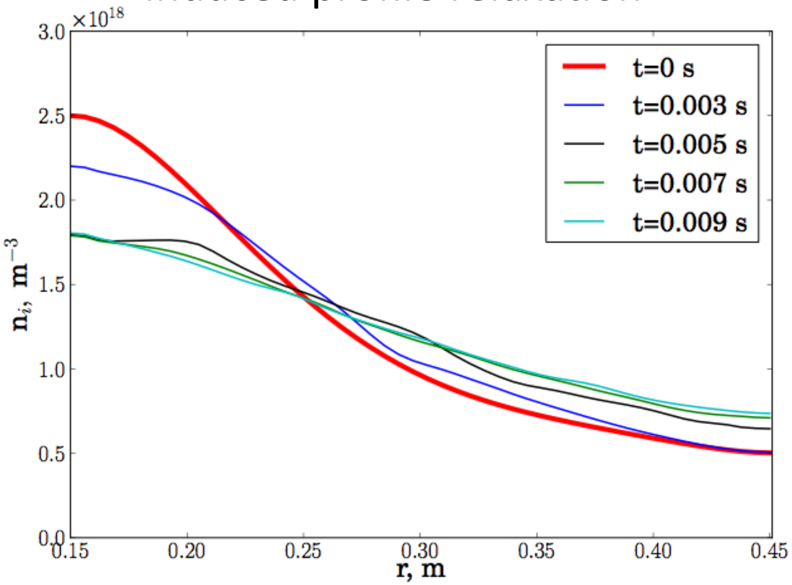
- Zero mean potential profile.
- Zonal flows evolved.



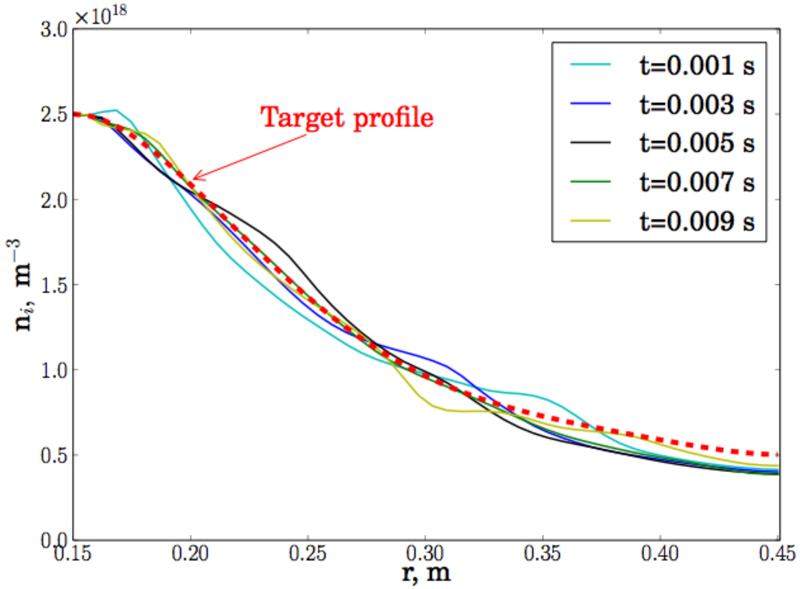


Density Profile Control Either Through Time Independent Source or Time Dependent Suppression of m=0

No source results in transport induced profile relaxation



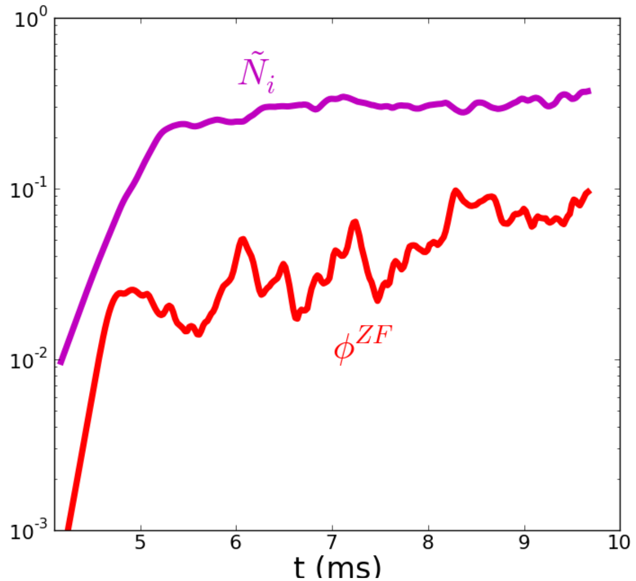
Time independent source partially controls finite m=0 fluctuation



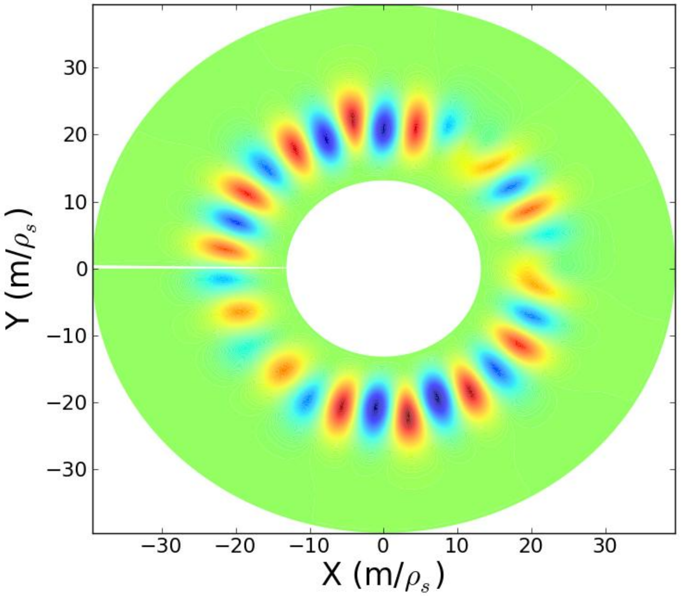
- Option 1: subtraction of m=0 density fluctuation at each time step.
- Option 2: evolve a source using the zonal density component. Integral part of PID source.
- Option 3: use the derived time averaged source from options 1 or 2 and use it as a time independent source. Shown in plot.

- No significant differences between the options except in the m=0 density fluctuation.

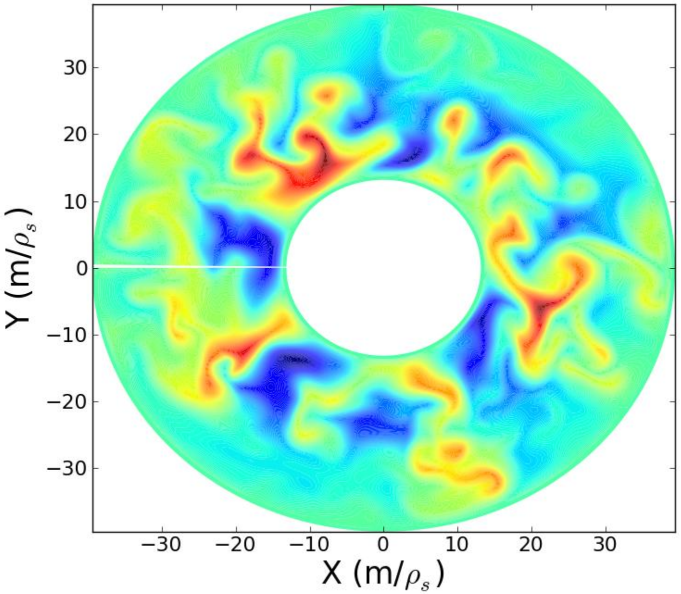
Nonlinear BOUT++ Simulations Grow by Linear Drift Wave Instability and Saturate by Nonlinear Interactions

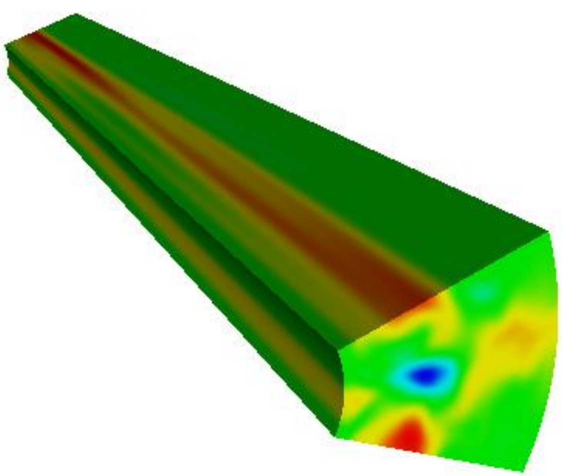


Most Unstable Linear Mode

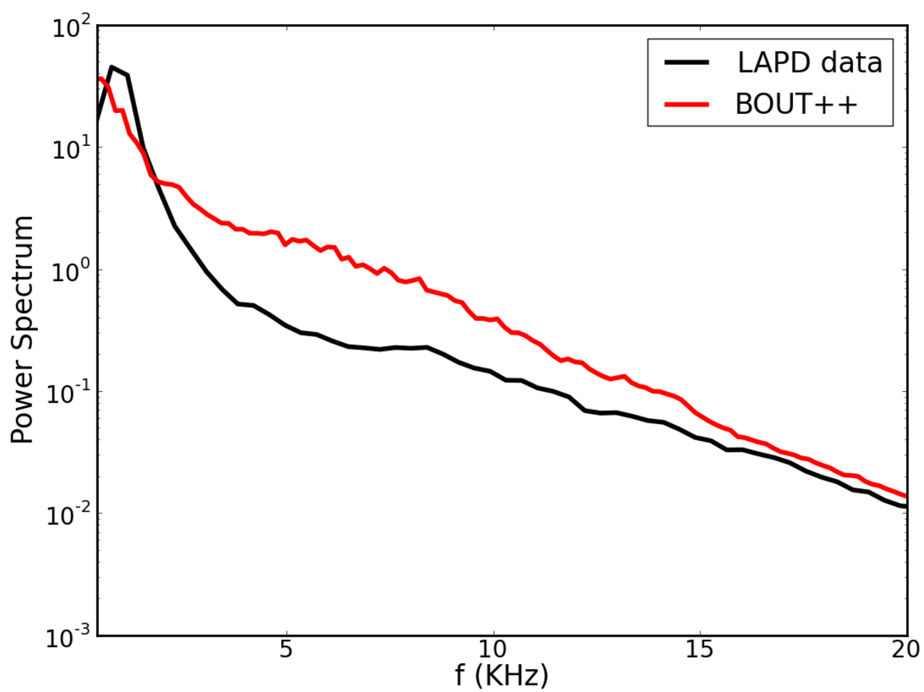


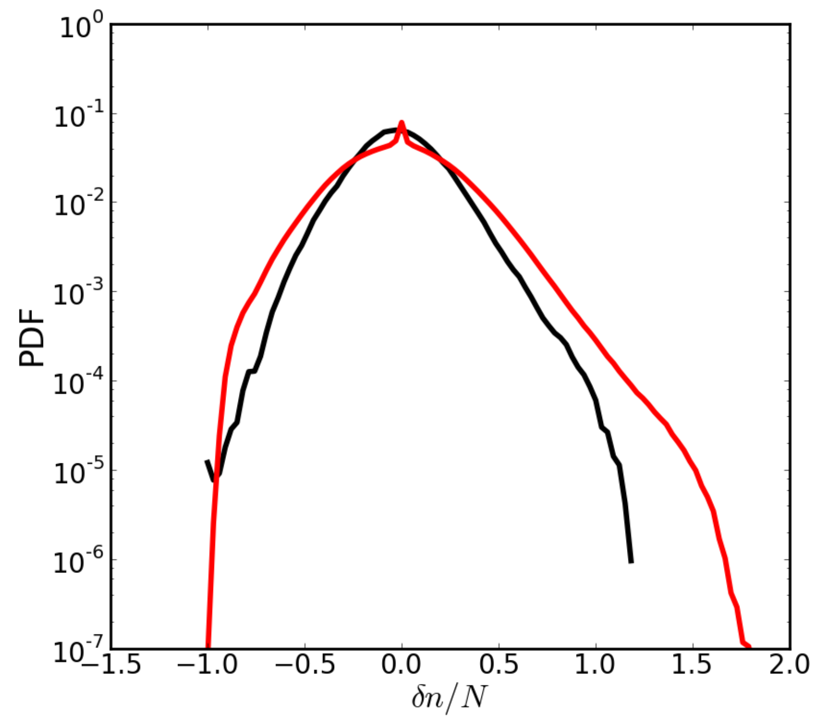
Non-Linear Saturated Turbulence





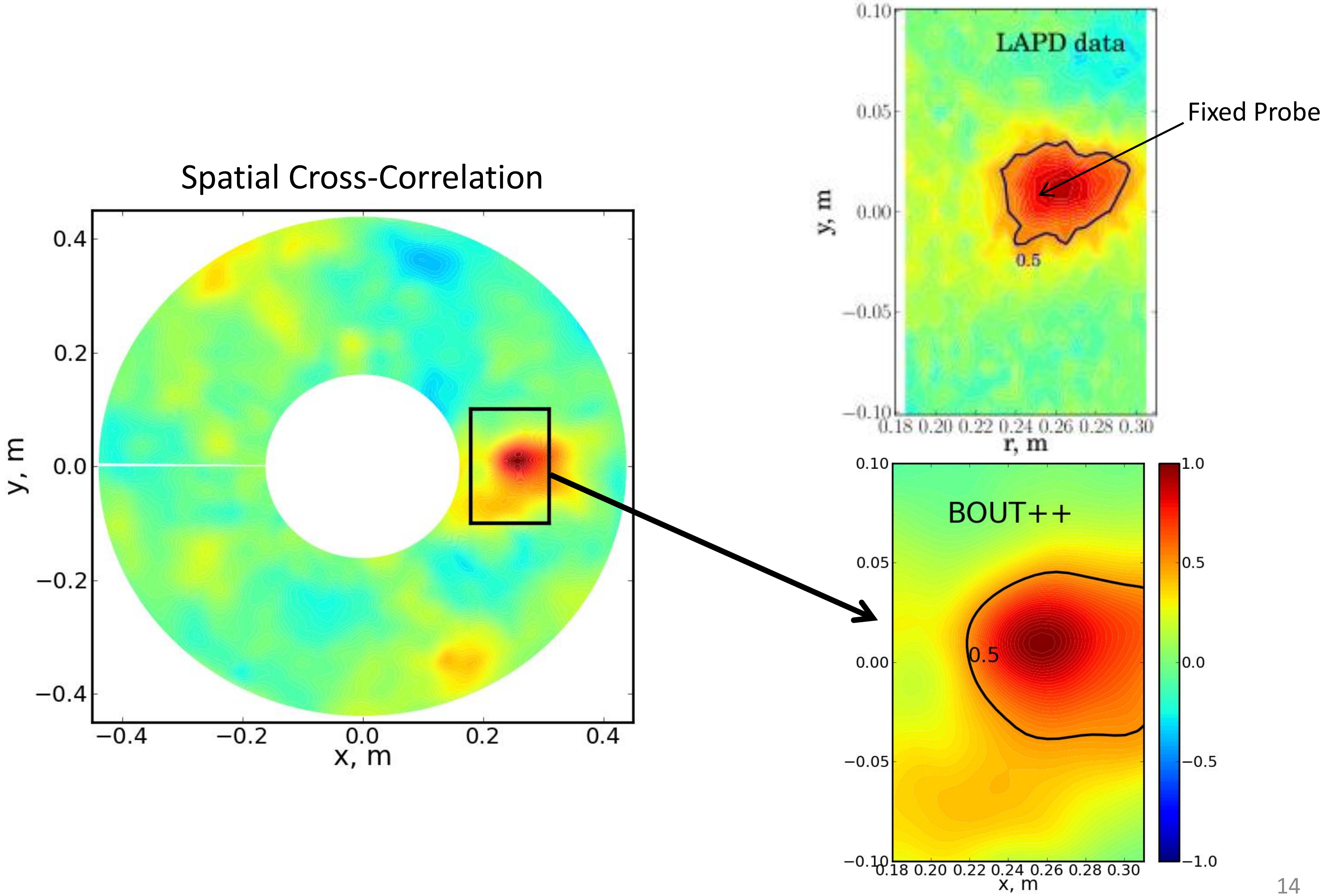
BOUT++ and LAPD Experimental Fluctuation Spectra Both Exponential at high Frequency but Have Different Slopes. PDFs Show Similar Non-Gaussian Features





Exponential spectra caused by Lorentzian-shaped temporal pulses where the width of the pulses sets the slope of the spectrum (D.C. Pace et al 2008).
Dissipation range spectra often exponential (P. Terry et al 2009).

BOUT++ Turbulence has Correlation Size A Few Times Larger than Experimental Turbulence



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Grid Convergence Study: Finite Difference Schemes Have Grid Spacing Dependent Errors

$$\left.\frac{\partial u}{\partial x}\right|_{x_i} = \text{(FD Scheme of order n)} \ + \underbrace{C(\Delta x)^n}_{\text{FD Error}} \left.\frac{\partial^{n+1} u}{\partial x^{n+1}}\right|_{x_i}$$

Dominant error in simulations due to first order upwind advection operator:

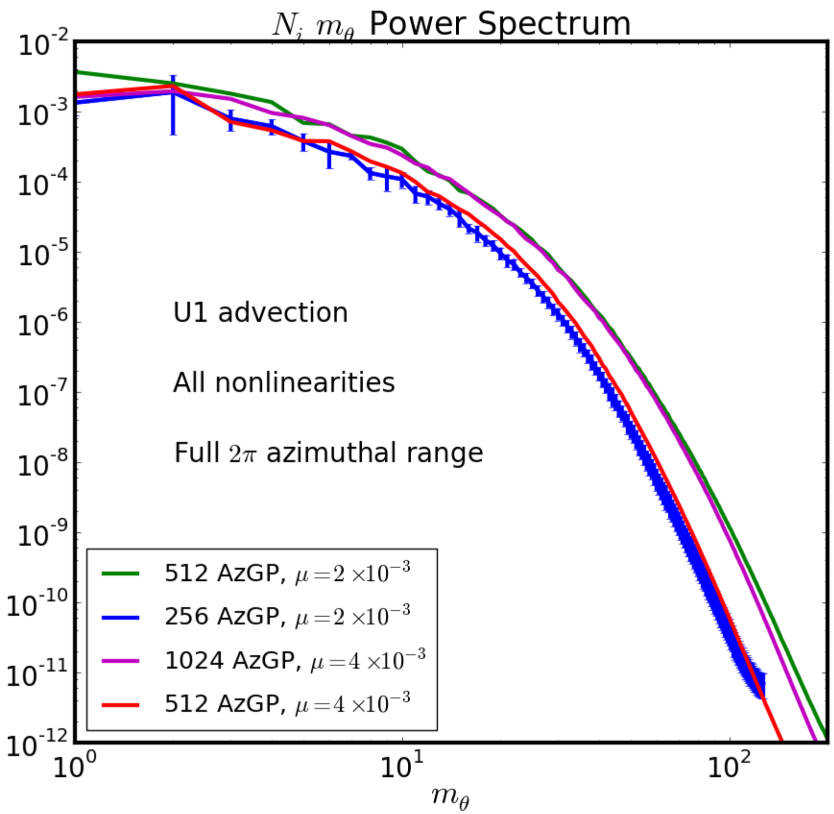
$$\mathbf{v_E} \cdot \nabla N \sim \frac{v_\theta}{r}\frac{\partial N}{\partial \theta} \quad \rightarrow \quad \text{Error: } \frac{v_\theta}{2r}\underbrace{\Delta x}_{\text{Diffusive error proportional to grid spacing}}\frac{\partial^2 N}{\partial \theta^2}$$

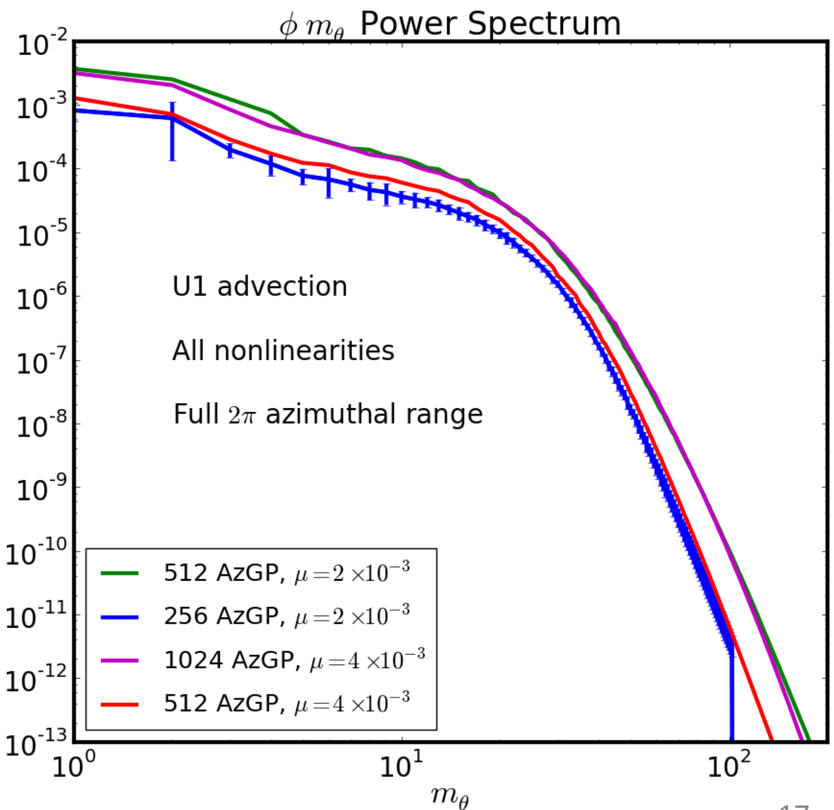
- Need fine grid spacing and/or higher order finite difference schemes to reduce the error and get better grid convergence.
- Problems are computational expenses and that the diffusive damping helps simulations saturate

Artificial Diffusive Operators Mimic Numerical Diffusion Errors in How They Change the Spectra

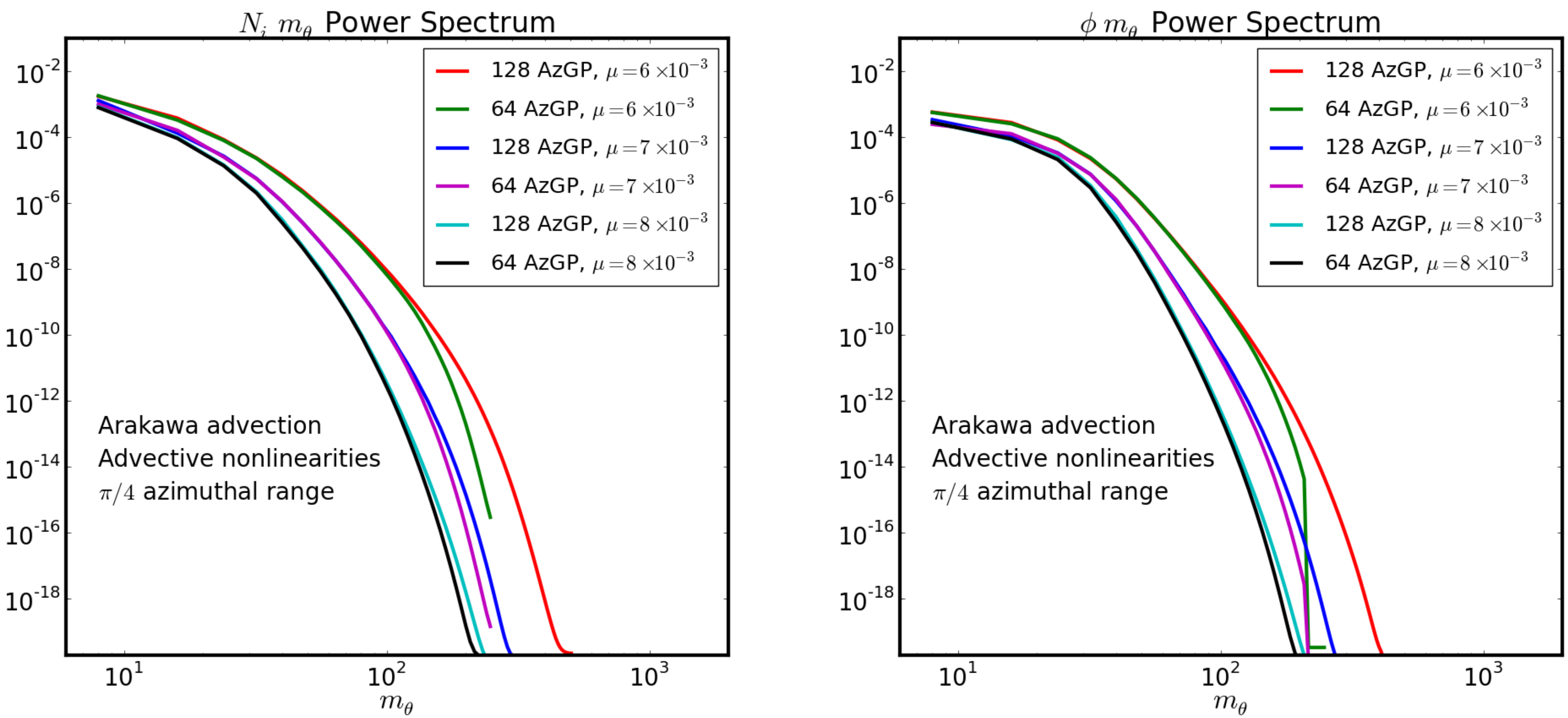
Artificial diffusion added to density equation: $\mu\nabla_\perp^2 N$

Artificial viscosity added to vorticity equation: $\mu\nabla_\perp^2 \varpi$





Better Grid Convergence is Achieved Using High Order Arakawa Advection Scheme



- Higher diffusion coefficient to get same saturation level
- Larger divergence at high k due to high order FD scheme error

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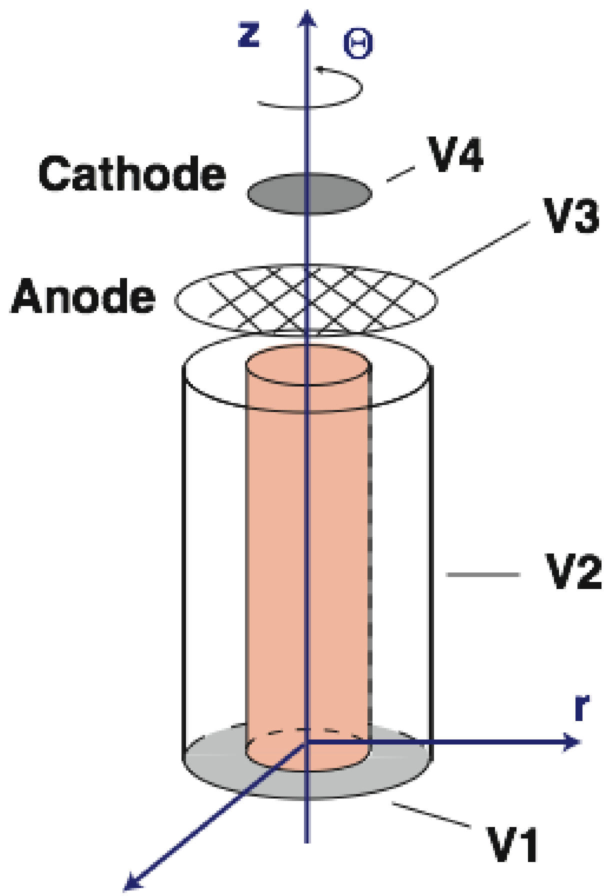
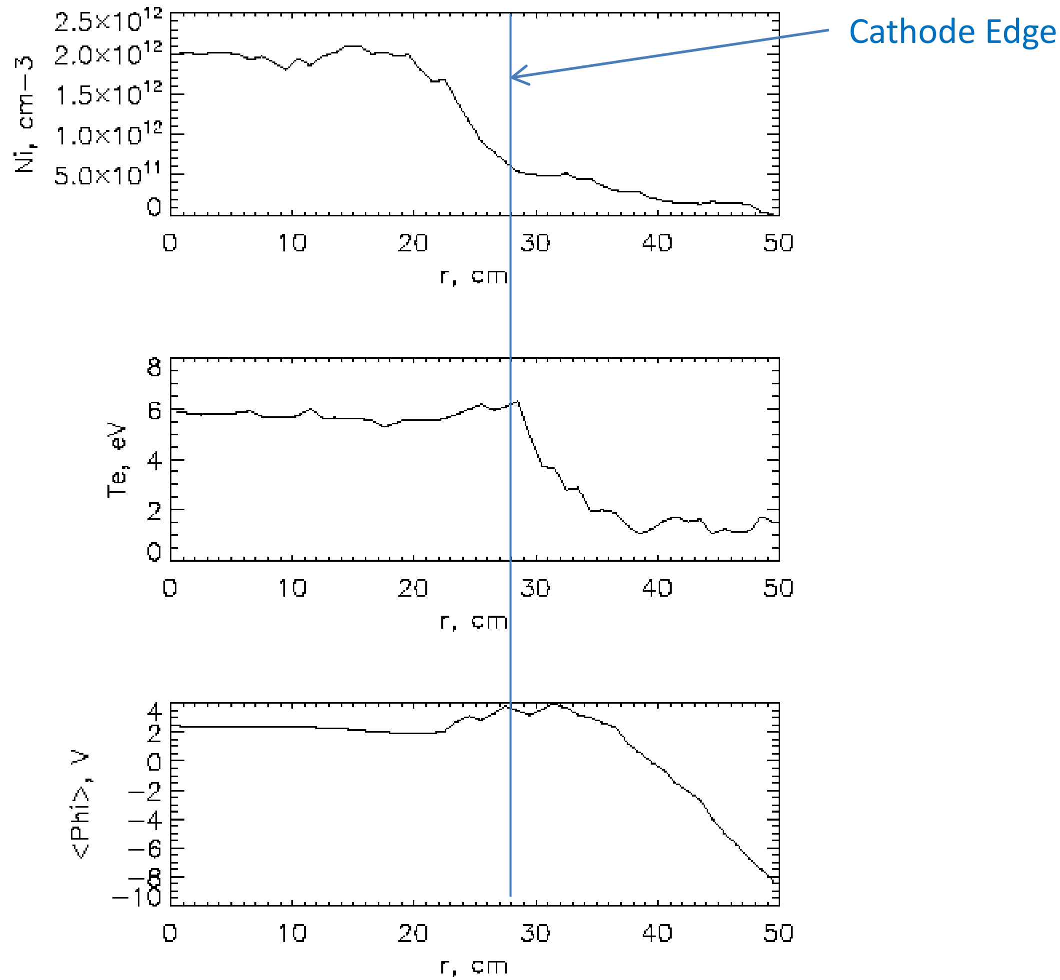
Boundary Conditions Important in Setting Flows and Temperature Profiles in LAPD

LAPD Boundary Schematic



Experimental Radial Profiles



Extensions of the Model

1. Axial Plate Sheath Boundary Conditions for Conducting Walls

$$j_{\|} = \pm eN \left[C_s - \frac{\sqrt{T_e/m_e}}{2\sqrt{\pi}} e^{(e\phi/T_e)} \right]$$

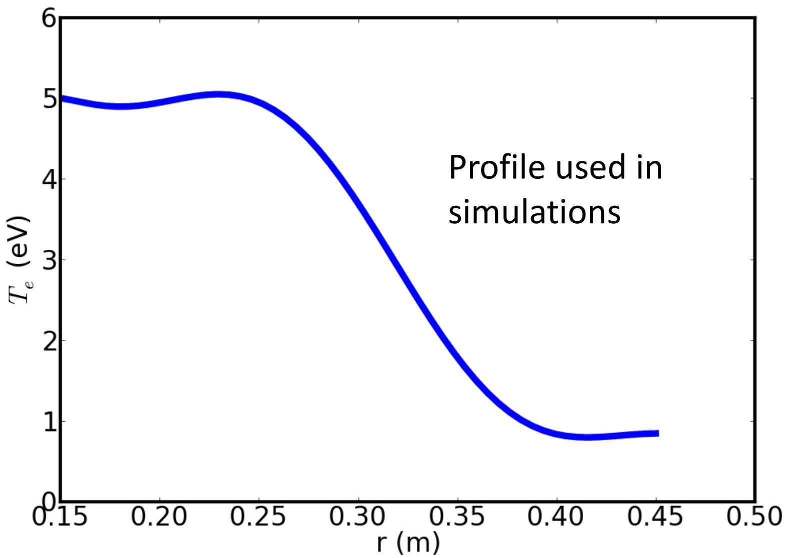
2. Heat transport equation

$$\frac{\partial T_e}{\partial t} = -\mathbf{v_E} \cdot \nabla T_e - v_{\|e} \nabla_{\|} T_e + 0.71 \frac{2}{3} \frac{T_e}{eN} \nabla_{\|} j_{\|}$$

$$+ \frac{2}{3N} \nabla_{\|} (\kappa_{\|e} \partial_{\|} T_e) - \frac{2}{3} T_e \nabla_{\|} v_{\|e} + S_T$$

Source used to subtract m=0 fluctuation component (same as density)

3. Equilibrium electron temperature profile from experiment



Sheath Implementation Tested in BOUT++ with the Conducting Wall Mode Instability[1]

Three-Field Model[2]

$$\frac{\partial \varpi}{\partial t} = \nabla_{\|} j_{\|}$$

$$\frac{\partial v_{\|e}}{\partial t} = \nabla_{\|}\phi - 0.51\nu_{ei} v_{\|e}$$

$$\frac{\partial T_e}{\partial t} = -\mathbf{V_E} \cdot \nabla T_e$$

Linearized Parallel Sheath Boundary Conditions

$$j_{\|} = \pm e N_o C_s (\Lambda_1 \phi + \Lambda_2 T_e)$$

Theoretical Values: $\Lambda_1 = 1, \Lambda_2 = log\sqrt{\frac{4\pi m_e}{m_i}}$

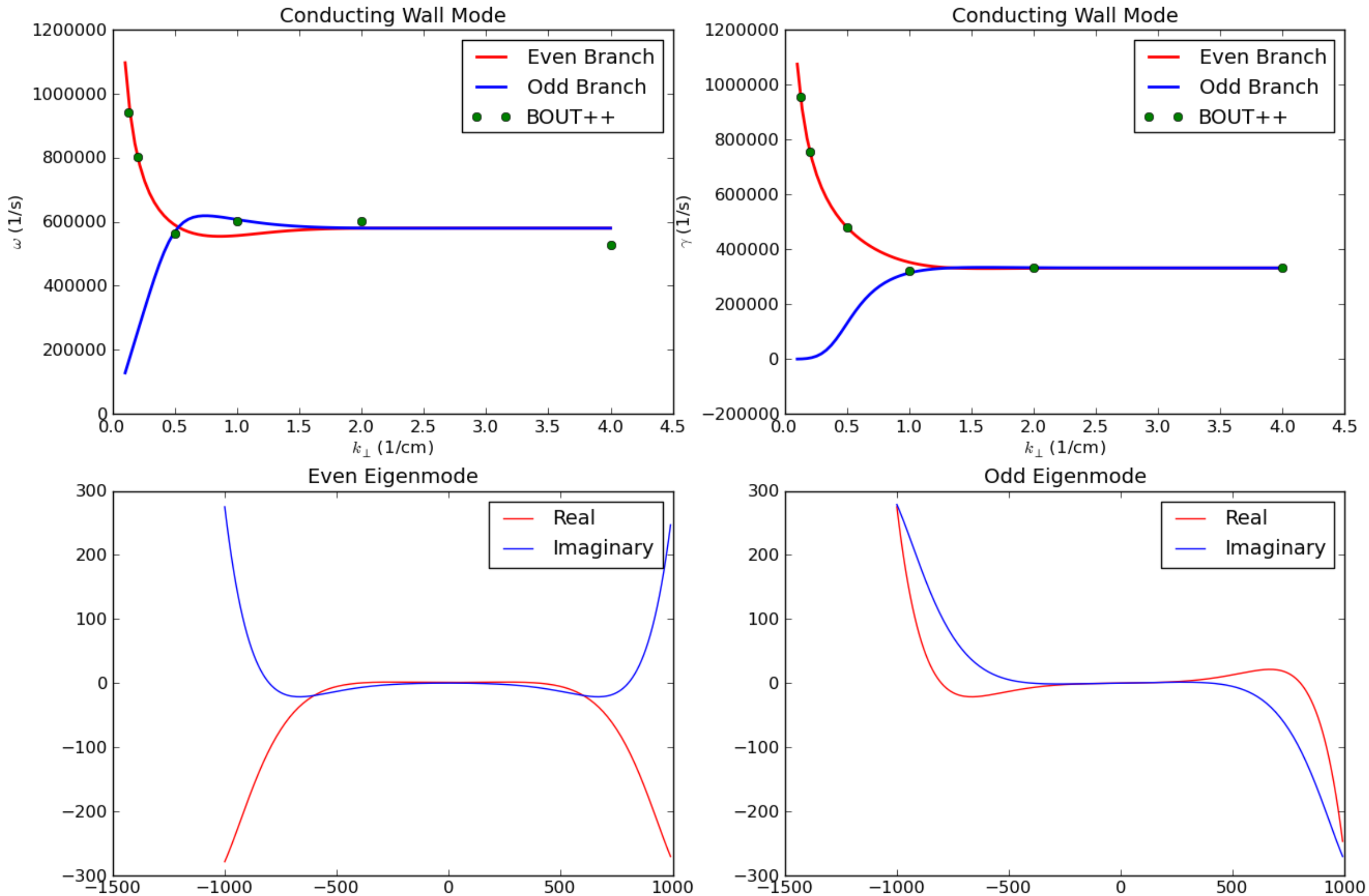
- Linear dispersion relation is a transcendental equation that can be solved numerically
- Infinite number of even and odd modes
- Python code numerically solves the transcendental equations for fastest growing even and odd modes

[1] Berk et. al. 1991
[2] Umansky, notes

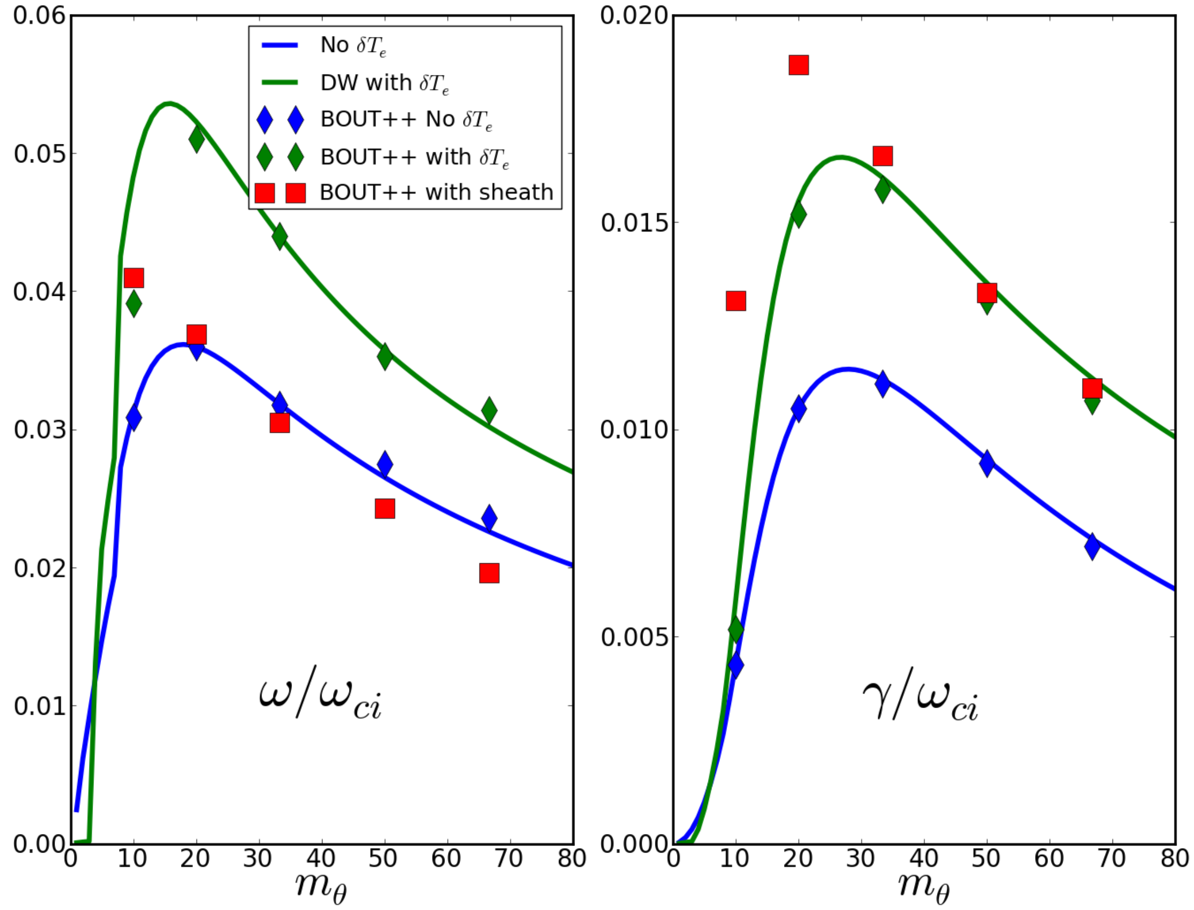
Simple Test Case Comparison Between Numerical Dispersion Relation Solver and BOUT++

Slab geometry, flat density profile, exponential temperature profile

$$\Lambda_1 = 0, \Lambda_2 = 1$$



Temperature Fluctuations and Sheath Boundaries Lead to Higher Growth Rates for Linear Drift Waves with LAPD Parameters and Profiles



- LAPD profiles and geometry. No potential profile.
- Four field linearized model with density and electron temperature gradient driven drift waves for green and red data.
- Three field model for blue data
- Lines show solutions using 1D radial eigenvalue solver program ([1]Eigsolver)
- Sheath problem is necessarily 2D so neither analytic nor eigsolver solution possible.

[1] Popovich and Friedman

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Conclusions, Works in Progress, and Future Work

- BOUT++ is a highly developed framework that allows for fairly easy coding of fluid models with the ability to add complex features.

- Linear machines like LAPD are ideal for use of fluid models.

- A large ongoing and relatively successful effort to validate a fluid model of LAPD turbulence has been conducted using BOUT and BOUT++.

- Advanced analysis of the simulation turbulence has been done and is still in progress and a few papers will be published soon.
    - Energy dynamics
    - The role of stable eigenmode branches
    - Blobs, filaments, and transport

- Future additions to the model include realistic axial boundary conditions, more fields, and possibly an equilibrium radial electric field. Start with a reduced model and add as needed.

- Desirable to compare a gyrofluid model to the results of the fluid model. Clarify the importance of kinetic effects in LAPD.